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A RUDIMENTARY OVERVIEW OF THE CAPABILITIES AND PROBLEMS CONCERNING THE FINITE-ELEMENT METHOD

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ABSTRACT

The finite-element method is very powerful and flexible to model complex geometries with computers, because the region of interest can be subdivided into finite elements accurately. The advantages and drawbacks of the finite-element method will be the focus of the rudimentary investigation presented in this report. Especially, the requirements in terms of computational effort and computer memory storage will be investigated with respect to Nuclear ElectroMagnetic Pulse (NEMP) research requirements, i.e., with configurations frequently found in NEMP research such as coupling and interaction studies, simulator design and sensor design. Such configurations are often three-dimensional and of intricate geometry.

It is found that with present day computer capability it is not yet possible to solve real-life three-dimensional geometries with the finite-element method, because of the memory requirements needed to store the resulting system of equations.

Two-dimensional geometries can at present be solved with the finite-element method, but the usefulness of two-dimensional geometries for NEMP research purposes is questionable.

RÉSUMÉ

Les avantages et inconvénients de la méthode des éléments finis sont présentés dans ce rapport. Plus particulièrement, les exigences en termes d'utilisation de mémoire et de temps d'ordinateur sont étudiées plus en détails. Cette étude est surtout orientée vers l'utilisation de la méthode appliquée à la recherches des effets des impulsions électromagnétiques (IEM), tels que l'étude des interactions électromagnétiques, de capteurs de champ électromagnétique et de simulateurs IEM. De tels problèmes sont généralement représentées par une géométrie tridimensionnelle très complexe.

Il a été trouvé qu'il n'est pas possible de résoudre des problèmes tridimensionnels complexes en utilisant la méthode des éléments finis; ces problèmes excèdent la capacité de mémoire des ordinateurs actuels.

Il est toutefois possible de résoudre des problèmes bidimensionnels par cette méthode; son utilité pour l'étude de problèmes IEM est par contre très limitée.

EXECUTIVE SUMMARY

At DREO, a number of well-known methods have been used to solve complicated electromagnetic scattering problems, which occur in Electromagnetic Pulse (EMP) analyses. We mention the Method of Moments and Transmission Line analyses. Most of these methods work in the frequency domain, and have certain disadvantages such as crude modelling capability and/or require prohibitively large computer resources and computer time. Therefore, the quest for better suited methods for EMP analyses continues.

More and more researchers employ the finite-element method to solve complicated scattering configurations. The finite-element method has unsurpassed modelling capabilities and looks promising for EMP research. The advantages and drawbacks of this relatively new method are presented in this report.

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1 INTRODUCTION

The finite-element method has been used in mechanical engineering for many years but was first applied to electromagnetics only 15 years ago. It is characterized by an intricate formulation, but the finite-element method is very powerful and flexible to model complex geometries with computers because the region of interest can be subdivided into finite elements accurately.

Probably the first book on the finite-element method applied to electromagnetics was written by Silvester [5] in 1980. Before that, articles on electromagnetic applications of the finite-element method have appeared in the literature since 1975. At first, the method was used to solve eddy-current problems and related problems. These applications allowed two-dimensional finite-element formulations based upon Maxwell's equations and neglected the displacement current (quasi-static solutions), thereby decreasing the complexity of the problem to be solved. The finite-element formulation was obtained using a variational principle, i.e., based on the Rayleigh-Ritz principle (Steele [3], p. 68).

Since 1984, researchers in electromagnetic disciplines other than power engineering have become more and more interested in the finite-element method because of its powerful properties. But many electromagnetic disciplines require that Maxwell's equations are solved without neglecting the displacement current. As examples we mention exploration geophysics (Stam [2]) and medical applications (hyperthermia). Modern finite-element formulations (in the time as well as in the frequency domain), which solve the full equations of Maxwell, are usually based on a Galerkin approach, i.e., the method of weighted residuals[2)] with the same weighting and basis functions.

The advantages and drawbacks of the finite-element method will be the focus of the rudimentary investigation presented in this report. Especially, the requirements in terms of computer time and computer memory storage will be investigated with respect to NEMP research requirements, i.e., with configurations frequently found in NEMP research such as coupling and interaction studies, simulator and sensor design. Such configurations are often three-dimensional and of complicated geometry. One usually prefers time-domain formulations, because the large bandwidth of the NEMP spectrum requires that the configuration must be solved for many frequencies. Hence, time-domain type of formulations tends to require less computational time than frequency-domain formulations.

So, the principal intention of this report is to try to answer the question:

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2) The Method of Moments is a one-dimensional method of weighted residuals, and can be seen as a one-dimensional finite-element method. See Harrington [6].

Can the finite-element method be used for NEMP research purposes?

Since we have just mentioned what typical NEMP research configurations are, this question can be reformulated more universally as:

Can the finite-element method be used to solve complicated three-dimensional configurations?

Of course, the answer to this question is subject to the limitations the current state of computer technology offers.

2 THE FINITE-ELEMENT METHOD

As mentioned in the introduction, the finite-element method is capable of solving complicated geometries by subdividing the region of interest in finite elements. Compared with for example the finite-difference method, it is not restricted to a particular grid structure.

2.1 The finite-element formulation

For two-dimensional problems the finite element often used is a triangle or quadrilateral segment and in three-dimensional problems an isoparametric hexahedron. Fig. 2.1 depicts some of these finite elements.

With the weighted-residual approach, the unknown field quantities to be solved are projected on a basis of known basis functions, i.e., the field quantities are written in terms of known basis functions with coefficients to be determined later. These usually linear basis functions span over each finite element. The basis function pertaining to a node of a specific finite element has unity magnitude in that node-point only. Such a node is called the supporting node of the basis function. At every node other than the supporting node this basis function vanishes.

Finally, the finite-element formulation is obtained by weighting the residual of the pertinent differential-equation over each finite element to zero with a suitable weighting function. With the Galerkin-type approach the weighting function is the same as the basis function. The above described procedure yields a sparse[3)] system of equations where the unknown coefficients, which result from the expansion of the unknown field quantities in basis functions, have to be solved for. In later sections it will be shown that storing and solving this system of equations is the main problem with the application of the finite-element problem.

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3) A matrix with few non-zero elements.

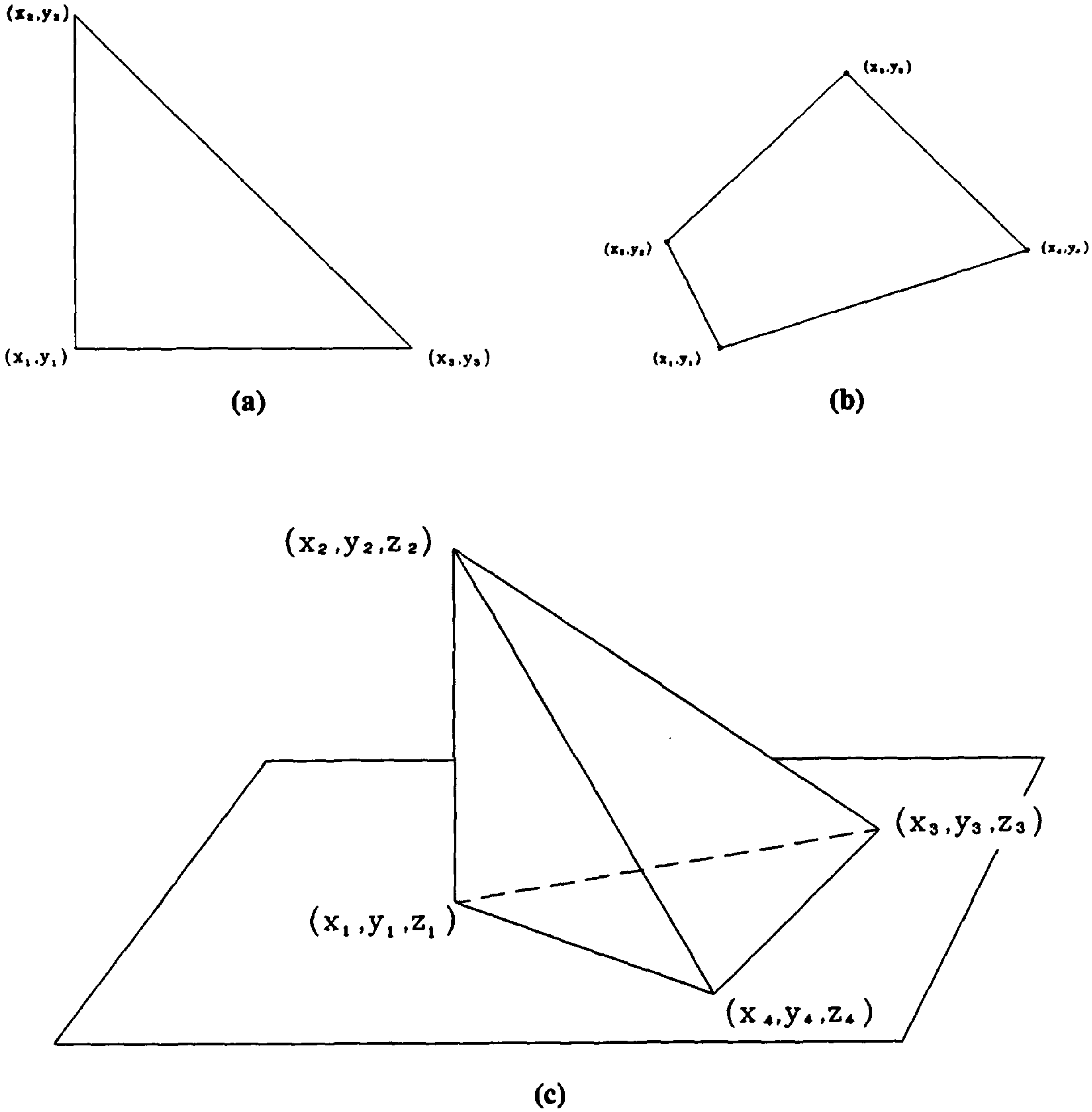

Fig. 2.1. Some often used finite elements.
a) 2D triangle finite element
b) 2D quadrilateral finite element
c) 3D hexahedron finite element

2.2 Boundary conditions

At a boundary between finite elements with different electromagnetic properties, e.g., a boundary between free space and a scattering object or between two adjacent finite elements in a strongly inhomogeneous medium, the basis functions and finite elements have to be constructed so that the boundary conditions are satisfied—see pp. 34 - 38 Stratton [7]. The tangential parts of the electric field and the magnetic flux density must be enforced to be continuous across such adjacent finite elements and the normal parts may jump by a finite amount. In Mur [8] and [9], Mur describes finite elements and basis functions which preserve the boundary conditions. Mur calls his finite elements "boundary elements." Most finite elements used in the literature or in commercially available computer programs do not preserve the boundary conditions.

2.3 Interior and exterior problems

In general, the problem to be solved can be divided in two different kind of problems: an interior or exterior problem. An interior problem is a configuration where the region of interest is bounded, like waveguides, while with an exterior problem it is not. The latter is the more difficult one to solve because the unbounded region of interest has to be restricted in size as much as possible, because of available computer memory.

The truncation of the unbounded region of interest into a bounded problem domain which is sufficiently small in size that it can be stored in the computer's memory yields an artificial outer boundary. This boundary is also known as the numerical domain boundary. Appropriate boundary conditions have to be applied upon this boundary in such a way that the fields in the configuration are not influenced. When the sources generating the incident field are located within the problem domain, and as long as the fields have not yet reached the numerical domain boundary, zero-valued boundary conditions for the field quantities have to be applied. When the sources are located outside the problem domain, known prescribed boundary conditions have to be enforced upon the pertinent differential equations. Again as long as the fields inside the problem domain have not yet reached the numerical domain boundary. Usually the time window is then too small. To circumvent this, either the size of the problem domain has to be increased—thereby the number of finite elements usually becomes prohibitively large—or appropriate boundary conditions have to be applied which take into account the electromagnetic radiation through the numerical domain boundary.

Applying these appropriate boundary conditions efficiently is at present still a subject of research. It is known as the problem of "absorbing boundary conditions." This problem is not unique for the finite-element method; it also occurs with the finite-difference method.

2.4 Subdivision of the problem domain

The number of finite elements required to model the problem domain depends strongly on:

- the geometric complexity of the region of interest and its boundaries,
- the electromagnetic properties of the materials,
- the desired time resolution.

Obviously, the more complex the geometry of the configuration, the more finite elements are needed.

Costache (pp. 13 - 15, Costache [1]) found that for two-dimensional highly-lossy media, e.g., a copper or metal sheet, the number of finite elements required to model the sheet in the frequency domain is skin depth dependent. As the skin depth decreases, it becomes harder for the (linear) basis functions associated with the finite elements in the sheet to follow the field variations. For accurate results the size of the finite elements must be at least ten times the skin depth. For time-domain problems one expects that an analogous rule exists.

Another factor which determines the required number of finite elements is the desired time resolution Δt. Let h denote the size of the smallest finite element of the mesh. We then require that

$$\Delta t \leq \frac{h}{c_0\sqrt{3}}.$$

This condition is known as the Courant condition. So, an additional restraint for the time-domain case is that the size of the smallest finite element must be larger than the distance a wavefront propagates in $\sqrt{3}\,\Delta t$ seconds.

3 COMPUTER REQUIREMENTS FOR A TIME-DOMAIN FINITE-ELEMENT METHOD[4)]

The Maxwell equations allow a representation for the electric and magnetic fields, which reduces the six unknowns (for a three-dimensional problem) to four by introducing a vector potential $\underline{A}$ (three unknowns) and a scalar potential ϕ (one additional unknown)—see pp. 23 - 34 Stratton [7]. Now, let the number of finite elements in which we subdivide the problem domain be denoted by N. For a three-dimensional problem the hexahedron finite-element, which has eight nodes, is often used. The number of unknowns is then $4 \times 8 \times N = 32\ N$. The resulting system of equations to be solved is a $32\ N \times 32\ N$ system. To store this matrix equation with single precision (REAL *4 in FORTRAN) the minimum amount of computer memory required is $(4 \times 32\ N)^2$ bytes $= 16\ N^2$ Kb. For frequency-domain solutions the necessary amount of memory is twice as large, because the matrices of the system of equations contain complex valued elements. This analysis is a worst-case estimate.

So, for a volume subdivided in only 64 finite elements the storage requirement is 64 Mb, which demonstrates that even for a modest number of finite elements the storage space becomes prohibitively large. Consequently, only very simple three-dimensional problems without much detail can be solved.

Although the finite-element method yields a sparse matrix, a sparse-matrix equation-solver which could reduce the computer memory storage requirements significantly seems not to exist. It can be concluded because most articles on the finite-element method give two-dimensional results. When these articles present three-dimensional results, which is not very often, the examples are always very simple (often highly symmetric, which allows reduction of the number of unknowns). Even for such simple configurations the required computer time is quite considerable, Mur [8] and [9].

Two-dimensional geometries require far less computer memory storage, but still in the order of N^2.

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4) Most of the material presented in this Chapter is taken from Costache [1].

4 CONCLUSIONS

At present, some three-dimensional finite-element formulations are available. However, to be able to solve complicated three-dimensional electromagnetic scattering problems with the finite-element method, computers are required with memories and speeds which are at present not yet at hand. The problem lies with the enormous amount of memory required to store the resulting system of equations. Unless schemes are conceived to store this sparse matrix equation[5)] more efficiently, the finite-element method is expected not to be useful for solving real-life three-dimensional electromagnetic problems in the near future.

The computation time to solve three dimensional problems is high. This is mainly caused by the time needed to solve the large system of equations. Iterative techniques have the preference above direct methods, because they are usually faster. It may be possible to formulate finite-element solutions which exploit the benefits of parallel processing, so that the computational time can be decreased significantly.

Two-dimensional problems can be solved with presently available computers, but are of limited use for NEMP research.

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5) The main matrix has only a few non-zero elements.

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6) Although the title of this report does not show, this report studies penetration in cavities with the finite-element method.

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"Optimum Choice of Finite Elements for Computing Three-Dimensional Electromagnetic Fields in Inhomogeneous Media," IEEE Transactions on Magnetics, Vol. MAG-24, No. 1, January 1988, pp. 330 - 333.

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